A Defense of Platonism as an Account of Mathematical Truth

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PHIL 3870: Philosophy of Mathematics

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May 17, 2024

In philosophy of mathematics, platonism (also known as realism) is the view that for a proposition *A* to hold true in a system of numbers, it holds true because of the system of numbers and the relations between them. As William Tait points out in "Truth and Proof: The Platonism of Mathematics," some have argued that platonism is an unsuitable view of mathematical truth because, while propositions reference real abstract mathematical objects, justifications of their truth are in the form of written proof than of observation of or causal interaction with those objects. As Tait summarizes, platonism (a view he disagrees with), "holding in the system of numbers[,] is inapplicable because we have no direct apprehension of this structure."[1] Notably, in his paper "Mathematical Truth," Paul Benacerraf expounds a similar argument, in which a platonistic theory of mathematical truth cannot satisfy two important concerns that a foundational theory of mathematics should satisfy:

> It is my contention that two quite distinct kinds of concerns have separately motivated accounts of the nature of mathematical truth: (1) the concern for having a homogeneous semantical theory in which semantics for the propositions of mathematics parallel the semantics for the rest of the language, and (2) the concern that the account of mathematical truth mesh with a reasonable epistemology.[2]

It is Benacerraf's view that "almost all accounts of mathematical truth can be identified with serving one or another of these masters at the expense of the other."[3] Benacerraf uses platonism to give an example of an account of mathematical truth failing to satisfy both the semantic and epistemological constraints simultaneously (in particular, he argues, platonism offers a good semantic account of mathematics but a bad epistemological one). Firstly, language used in a platonistic account of mathematics can be applied in other non-mathematical contexts.

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[1] Tait, W. W. "Truth and Proof: The Platonism of Mathematics." Synthese 69, no. 3 (1986): 341. http://www.jstor.org/stable/20116347.

[2] Benacerraf, Paul. "Mathematical Truth." The Journal of Philosophy 70, no. 19 (1973): 661. https://doi.org/10.2307/2025075.

[3] Ibid.

To borrow an example from Benacerraf, the sentential structure "there are at least three *FG*'s that bear *R* to *a*" may be realized as "there are at least three large cities older than New York" just as much as it could be "there are at least three perfect numbers greater than 17" if we take the platonistic (and logicist) position that numbers such as 17 are real objects that exist outside of space and time and independently of the human mind.

Then, Benacerraf argues that the platonistic account has epistemological problems. First, such an account would need a theory of truth similar to the one Tarski provided for a language with predicates, singular terms, and quantifiers. But even then, one could still not possess mathematical knowledge under this "standard" account, for Benacerraf prefers a causal explanation of knowledge:

> It must be possible to establish an appropriate sort of connection between the truth conditions of *p* (as given by an adequate truth definition for the language in which *p* is expressed) and the grounds on which *p* is said to be known, at least for propositions that one must *come to know*—that are not innate. In the absence of this, no connection has been established between *having those grounds* and *believing a proposition which is true*. Having those grounds cannot be fitted into an explanation of *knowing p*. The link between *p* and justifying a belief in *p on those grounds* cannot be made. But for that knowledge which is properly regarded as some form of justified true belief, then the link *must* be made.[4]

In other words, if an individual has no causal relation to the objects, properties, and relations which a proposition is about, then they cannot be said to have knowledge regarding the proposition. For one to know a proposition, there must be a causal connection between the knower and that which the proposition is about. Such a connection would be impossible in the platonistic account of mathematical truth since the platonist claims that numbers exist outside of space and time, presumably free from any causal link to any person seeking mathematical truth. Therefore, by choosing the platonistic account because of its merits regarding Benacerraf's

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[4] Ibid., 672–73.

semantic horn, we sacrifice any causal connection to the contents of mathematics by which we might be able to claim knowledge.

William Tait responds to Benacerraf by positing that the platonist view does not actually have problems in the epistemological realm. Tait tells us to consider two sentences:

(1) There is a prime number greater than 10
(2) There is a chair in the room[5]

According to Tait, any argument claiming that one cannot have knowledge of (1) could equally apply to (2), so if we can claim (2) on the basis of observing a chair in the room, then we can claim (1) by proof. To see this, we might claim that our perception of the chair gives us grounds to claim (2) whereas a proof does not involve perception and thus does not give us grounds to claim (1). To this, Tait responds by asking "what have my experiences to do with physical objects and their relationships *at all*?"[6] After all, neither (1) nor (2) are statements about experiences or sensations, so why should we claim them on experiential or sensory grounds? Tait takes a Wittgensteinian approach, claiming that (1) and (2) both involve learning language and applying it as well as learning what makes it rational to use language in such a way. Therefore, while perceiving physical objects such as chairs is different from learning to prove mathematical propositions, in both cases, we must first learn what makes our use of language rational.

A greater obstacle Tait presents against Benacerraf is his claim that doing mathematics is simply not empirical:

> Consider a case of interdependence: a mathematical prediction of the motion of a physical object. First, we read the appropriate equations off the data—i.e., we chose [sic] the appropriate idealization of the phenomenon. Second, we solve the equations. Third, we interpret the solution empirically. When Benacerraf speaks of mathematical knowledge in his paper, the relevant kind of knowledge is knowledge that *S*, where *S* is a

[5] Tait, 342.

[6] Ibid., 343.

> mathematical proposition. But *that* kind of knowledge is involved only at the second step, and it involves nothing empirical. The first and third steps involve only knowing how to apply mathematics to the phenomena… The fact is that we do know how to apply mathematics and we do not causally interact with mathematical objects. Why doesn't this fact simply refute a theory of knowing how that implies otherwise?[7]

According to Tait, by virtue of knowing that mathematics applies to the physical world and by virtue of not causally interacting with mathematical objects, Benacerraf's causation criterion for knowledge must not hold for a platonistic account of mathematical truth. Furthermore, Tait believes Benacerraf to be misguided in that establishing a causal link between mathematical objects and the knower would not be enough to claim mathematical knowledge. We would additionally need to show "whether our canons of proof obtain their meaning and validity from such perceptions."[8]

While I believe that platonism actually does satisfy Benacerraf's semantic and epistemological criteria for mathematical knowledge, I don't think Tait properly responds to Benacerraf: rather than directly tackle the problem of a lack of causation, Tait bypasses it. Tait's stance is that perceptions (if there are any) obviously cannot contribute meaningfully to proofs in the same way perception of physical objects is not by itself sufficient for knowledge about physical objects. Tait argues that our canons of proof do not gain meaning and validity from perceptions by pointing out that we began to distinguish sets at a point in time. It seems to me that by pointing this out, Tait means to argue that the validity of a proof should not be different at different points of time. I think this is actually not a problem for the epistemological horn for platonism.The justification of (2) via perception presupposes an understanding of the concept of a chair which we end up perceiving. Similarly, the justification of (1) via perception presupposes

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[7] Ibid., 351.

[8] Ibid., 346.

an understanding of numbers; for the truth of (1) to even be in question requires us to have some understanding of numbers. So in a time before sets were distinguished, the truth of any statement about sets would not be under question, and once we did begin to distinguish sets, we could then use perception to prove the statement.

Although it is possible to show that perception could possibly give proof meaning or validity, it is hard to show that this must be the case. However, if the truth of a proof were to rely on such an assumption, I also don't think this would be an issue. In mathematics, we already have axioms and mathematical statements which rely on those axioms, meaning such statements would have to be taken with a grain of salt. Similarly, we could take mathematical statements with a grain of salt since their truth would necessitate that perception of mathematical objects give meaning to the truth of the statements (in our platonistic account of mathematical truth).

Our goal, then, would be to show that we do perceive mathematical objects. First we must make clear what it means to perceive a non-physical object. In the usual usage, when we say that we perceive things non-physical, we mean that we become aware of or recognize (such as perceiving events). Therefore, if we prove that we perceive mathematical objects when writing proofs, we prove a causal connection between mathematical objects and the knower which is sufficient to show that platonism works well both as a semantic account of mathematical truth and an epistemological one.

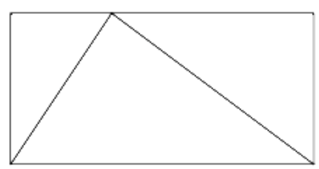
Thus, I argue that we perceive objects when proving mathematical propositions. Consider Plato's *Meno*, for instance, in which Socrates helps a slave construct a geometric proof. Plato claims that the slave recollected the proof from a previous life (and calls this anamnesis).[9] In proving that the area of one square is twice that of another, the slave must perceive triangles, squares, areas, and lines, and the steps involved in reaching the conclusion are also perceived

[9] Plato, *Meno*, 82b–85c.

(distinguished from a past life or some other source). Since the slave's proof consists of perceptions of mathematical objects, there is a causal link between the slave and the objects about which the slave is trying to prove. In fact, any "innate" (or in this case, received from a past life) beliefs or attitudes one might hold lie contrary to Benacerraf's position concerning a causal link between that outside of space and time and that within.

A similar route I will take to defend platonism in the epistemological sense involves taking a more mathematics-related approach similar to one a mathematician would put forth when encountered with Benacerraf's dilemma, having to defend their life's work from an issue which seemingly reduces their profession to epistemologically unsound.

Consider Paul Lockhart's consideration of a triangle's area, part of his argument in *A Mathematician's Lament* that mathematics is an art and should be taught accordingly in schools:



> I wonder how much of the box the triangle takes up? Two thirds maybe? The important thing to understand is that I'm not talking about this *drawing* of a triangle in a box. Nor am I talking about some metal triangle forming part of a girder system for a bridge. There's no ulterior practical purpose here. I'm just *playing*… Even the most carefully made physical triangle is still a hopelessly complicated collection of jiggling atoms; it changes its size from one minute to the next… The *mathematical* question is about an imaginary triangle inside an imaginary box. The edges are perfect because I want them to be—that is the sort of object I prefer to think about.[10]

While not a philosophical writing, Lockhart provides an interesting insight into the minds of those most central to the question of how mathematical knowledge is gained. When he uses the word "imaginary," we can assume Lockhart means existent but non-physical. More

[10] Lockhart, Paul. *A Mathematician's Lament* (New York, NY: Bellevue Literary Press, 2009), 24–25.

importantly, however, we see that proof for a mathematician involves “creating” mathematical objects. Under the platonic view, those objects would have already existed, meaning the mathematician was actually somehow summoning the object so that she could reason about it.[11] So far, we have not even begun to prove that a triangle’s area equals half of its base multiplied by its height, but we have already extracted geometric objects which we represent on paper (or a screen) and potentially reason about. In order for any thought to occur at all about the triangle, there must be some causal interaction between the thinker and the object (for example, the slave “creates” lines, which lead the slave to reason about the areas of squares). Importantly, the proof does not concern the actual symbols on the paper but rather what they represent. Proof, therefore, would not be a mental event as the intuitionists might suggest.

In conclusion, Benacerraf finds it puzzling that there does not seem to exist any account of mathematical truth or knowledge which can allow its semantics to extend beyond mathematics and be epistemologically valid. Benacerraf considers platonism, which works well semantically in non-mathematical situations but cannot purport to use proofs to show truth since truth statements about mind-independent objects must be empirical, which proofs are seemingly not. Tait, on the other hand, does not find it puzzling that no account satisfies Benacerraf’s two criteria because he thinks Benacerraf’s first criterion is not really a criterion and that Benacerraf is misguided in thinking that showing a causal connection between a person and mathematical objects shows epistemological soundness. Ultimately, though, Tait admits that there are truth values independent of the mathematician that may or may not be provable under certain

[11] Note that later in the proof, Lockhart “draws lines” similar to the slave in *Meno*. This would also be a case of “summoning” mathematical objects. Some proofs also “alter” lines or other objects. For example, proving the correctness of Djikstra’s algorithm involves maintaining a “frontier” heap which is always changed. Under the platonist account of mathematical truth, we incorrectly perceive these objects as changing, when really we stop “perceiving” the old object and start perceiving a new (already existing) one such that it appears that the old object turns into the new one.

properties.[12] I believe that Benacerraf's goal of finding a causal link between the mathematician and mathematical objects in order to find an account that satisfies both criteria is not misguided, but I disagree with Benacerraf when he states that platonism fails to do this. I believe that a platonistic account of mathematical truth is consistent with the current state of mathematics and how proofs are done with axioms and that there really is a causal link between the mathematician and the object with which she works.

[12] Tait, 363.

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